

# Nucleon decay via dimension-6 operators in the $E_6 \times SU(2)_F \times U(1)_A$ SUSY GUT model

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In our previous paper [N. Maekawa and Y. Muramatsu, Phys. Rev. D **88**, 095008 (2013)], we showed that  $R_1 \equiv \frac{\Gamma_{n \rightarrow \pi^0 + \nu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}}$  and  $R_2 \equiv \frac{\Gamma_{p \rightarrow K^0 + \mu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}}$  can identify the grand unification group  $SU(5)$ ,  $SO(10)$ , or  $E_6$  in typical anomalous  $U(1)_A$  supersymmetric (SUSY) grand unified theory (GUT) in which nucleon decay via dimension-6 operators becomes dominant. When  $R_1 > 0.4$  the grand unification group is not  $SU(5)$ , while when  $R_1 > 1$  the grand unification group is  $E_6$ . Moreover, when  $R_2 > 0.3$ ,  $E_6$  is implied. The main ambiguities come from the diagonalizing matrices for quark and lepton mass matrices in this calculation once we fix the vacuum expectation values of GUT Higgs bosons. In this paper, we calculate  $R_1$  and  $R_2$  in  $E_6 \times SU(2)_F$  SUSY GUT with anomalous  $U(1)_A$  gauge symmetry, in which realistic quark and lepton masses and mixings can be obtained though the flavor symmetry  $SU(2)_F$ , which constrains Yukawa couplings at the GUT scale. The ambiguities of Yukawa couplings are expected to be reduced. We show that the predicted region for  $R_1$  and  $R_2$  is more restricted than in the  $E_6$  model without  $SU(2)_F$ , as expected. Moreover, we re-examine the previous claim for the identification of grand unification group with 100–1000 times more model points ( $10^6$  model points), including the  $E_6 \times SU(2)_F$  model.  
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## 1. Introduction

Grand unified theory (GUT) [2] is one of the most favorable candidates for a model beyond the standard model (SM). It has advantages not only theoretically but also experimentally. Theoretical advantages are that it can unify the three gauge interactions in the SM into a single gauge interaction and particles in the SM into fewer multiplets. Experimental advantages are that measured values of the three gauge couplings agree with the predicted values in supersymmetric (SUSY) GUT and measured hierarchies of masses and mixings of quarks and leptons can be understood, if it is assumed that **10** matter induces stronger hierarchies for Yukawa couplings than  $\bar{\mathbf{5}}$  matter [3,4].

The nucleon decay [2,5,6] is one of the most important predictions in GUTs. In GUTs there are new colored and  $SU(2)_L$  doublet gauge bosons, which we call  $X$ -type gauge bosons. In  $SU(5)$  GUT models these gauge bosons are  $X(\bar{\mathbf{3}}, \mathbf{2})_{\frac{5}{6}}$  and  $\bar{X}(\mathbf{3}, \mathbf{2})_{-\frac{5}{6}}$ , where  $\bar{\mathbf{3}}$  and  $\mathbf{2}$  means the antifundamental representation of  $SU(3)_C$  and the fundamental representation of  $SU(2)_L$ , respectively, and  $\frac{5}{6}$  means the hypercharge. Exchanges of the  $X$ -type gauge bosons induce dimension-6 operators, which break both the baryon and lepton numbers and induce the nucleon decay. Usually, the main decay mode of the proton via dimension-6 operators is  $p \rightarrow \pi^0 + e^c$ . The mass of  $X$  is roughly equal to the GUT

scale at which three gauge couplings in the SM are unified into a single gauge coupling  $g_{\text{GUT}}$ , and therefore the lifetime of the nucleon can be estimated. In the minimal SUSY GUT model, the GUT scale  $\Lambda_G$  is  $2 \times 10^{16}$  GeV, and therefore the lifetime can be estimated as roughly  $10^{36}$  years, which is much larger than the current experimental lower bound,  $10^{34}$  years [7].

The triplet (colored) Higgs, which is the GUT partner of the SM doublet Higgs, also induces nucleon decay. Because of the smallness of Yukawa coupling for first- and second-generation matters, the constraint on the triplet Higgs mass from the experimental limits of the nucleon lifetimes is not so severe without SUSY. However, once SUSY is introduced, this constraint becomes severe, because this induces nucleon decay via dimension-5 operators [6]. In the minimal  $SU(5)$  SUSY GUT model, the lower bound for the triplet Higgs mass becomes larger than the GUT scale  $\Lambda_G$  [8,9], if the SUSY breaking scale is around 1 TeV.

The constraint on the triplet Higgs mass gives one of the most difficult problems in SUSY GUTs, i.e., the doublet–triplet splitting problem. The SM doublet Higgs mass must be around the weak scale to realize electroweak symmetry breaking, while, as noted above, its GUT partner, the triplet Higgs must be heavier than the GUT scale. Of course, we can realize such a large mass splitting by fine-tuning; however, this is unnatural. A lot of attempts have been proposed to solve this problem (for a review, see Ref. [10]). However, in most of the solutions, some terms that are allowed by the symmetry are just neglected, or the coefficients for some terms are taken to be very small. Such requirements are, in a sense, fine-tuning, and therefore, some mechanism that can realize such a large mass splitting in a natural way is required. One way to solve this problem is to introduce an infinite number of fields in 4D spacetime for the realization of the stability of the nucleon and the mass splitting. Several mechanisms have been proposed by introducing non-compact family gauge symmetry [11], an extra dimension [12–15], or strong dynamics for breaking the GUT symmetry [16].

The doublet–triplet splitting problem can be solved in 4D GUT under a natural assumption by introducing anomalous  $U(1)_A$  gauge symmetry. The natural assumption means that all interactions that are allowed by symmetries of the models are introduced with  $O(1)$  coefficients [17–21]. Higher-dimensional interactions are also introduced if they are allowed by the symmetries. One of the most interesting predictions of anomalous  $U(1)_A$  SUSY GUT models is that nucleon decay via dimension-6 operators becomes dominant [18,19]. In these models the gauge coupling unification requires that the cutoff  $\Lambda$  must be the usual SUSY GUT scale  $\Lambda_G$  and the real GUT scale  $\Lambda_u$  is

$$\Lambda_u \sim \lambda^{-a} \Lambda_G, \quad (1)$$

where  $\lambda < 1$  is the ratio of the Fayet–Iliopoulos (FI) parameter to the cutoff  $\Lambda$ . Because the anomalous  $U(1)_A$  charge of the adjoint Higgs  $a$  is negative,  $\Lambda_u$  is smaller than  $\Lambda_G$ ; therefore, nucleon decay via dimension-6 operators is enhanced. On the other hand, nucleon decay via dimension-5 operators is strongly suppressed [17–19]. Therefore, nucleon decay via dimension-6 effective operators is important in this scenario. One more important feature is that realistic quark and lepton masses and mixings can be realized in anomalous  $U(1)_A$  SUSY GUT models, with  $SO(10)$  and  $E_6$  grand unification group [17,20].

In the previous paper [1], we have calculated various partial decay widths of the nucleon from the effective dimension-6 interactions in the anomalous  $U(1)_A$  SUSY GUTs with  $SU(5)$ ,  $SO(10)$ , or  $E_6$  grand unification group. The predicted lifetime becomes just around the experimental lower bound, though the lifetime is strongly dependent on the explicit GUT models and the parameters. Therefore, it could happen that nucleon decay will be detected soon. Nucleon decay would be a good target for a future project. It is difficult to kill the anomalous  $U(1)_A$  GUT models from the limit of the lifetime

because the lifetime is proportional to the unification scale to the fourth. However, we have claimed that the identification of the unification group in the anomalous  $U(1)_A$  GUT scenario is possible if several partial decay widths can be measured. The ratio  $R_1 \equiv \frac{\Gamma_{n \rightarrow \pi^0 + \nu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}}$  is useful to identify the size of the rank of the unification group because the contributions from the new  $X$ -type gauge bosons  $X'$  in  $SO(10)$  and  $X''$  in  $E_6$  make  $R_1$  larger generically [22,23]. Also, the ratio  $R_2 \equiv \frac{\Gamma_{p \rightarrow K^0 + \mu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}}$  is useful to catch the contribution from  $X''$ , which is mainly coupled with the second-generation fields of  $\bar{\mathbf{5}}$ . Note that these ratios are not dependent on the absolute values of vacuum expectation values (VEVs) of GUT Higgs bosons. However, the results are strongly dependent on the mass ratios of  $X$ -type gauge bosons. It is important that the contribution from the extra gauge multiplet  $X'$  always becomes sizable in anomalous  $U(1)_A$  GUT because the mass of  $X'$  becomes almost the same as the mass of the  $SU(5)$  superheavy gauge multiplet  $X$ . The contribution from  $X''$  can be large, though it is dependent on explicit models. As a result, the identification becomes possible by measuring the ratios  $R_1$  and  $R_2$ . Once the masses of  $X$ -type gauge multiplets are fixed, the main ambiguities come from the diagonalizing matrices of Yukawa matrices. These ambiguities cannot be fixed only from measured masses and mixings of quarks and leptons, because we have a lot of  $O(1)$  coefficients in the anomalous  $U(1)_A$  GUT. If we would like to predict more concrete values for the various nucleon decay modes, we must fix these  $O(1)$  coefficients.

If we introduce the family symmetry  $SU(2)_F$  into the anomalous  $U(1)_A$  GUT with  $E_6$  unification group, the model predicts a characteristic scalar fermion mass spectrum in which the third-generation  $\mathbf{10}_3$  of  $SU(5)$  can have different universal sfermion masses  $m_3$  from the other sfermions, which have universal sfermion masses  $m_0$  [24–27]. If we take  $m_0 \gg m_3$ , the SUSY flavor-changing neutral current (FCNC) problem can be improved without destabilizing the weak scale, because the FCNC constraints are weakened for large first two-generation sfermion masses  $m_0$  while the stop masses  $m_3$ , which are important for stabilization of the weak scale, can be around the weak scale. In addition, if the CP symmetry is imposed, which is spontaneously broken by the Higgs that breaks  $SU(2)_F$ , not only can the SUSY CP problem be solved, but also the number of  $O(1)$  coefficients for quark and charged lepton masses and quark mixings can be smaller than the number of these mass and mixing parameters [28–31]. This means that the diagonalizing matrices can, in principle, be fixed from the quark and lepton masses and mixings, at least at the GUT scale. In Refs. [28,29], it has been shown that the quark and charged lepton masses and the Cabibbo–Kobayashi–Maskawa (CKM) matrix [32,33] can be consistent with the values evaluated at the GUT scale in the minimal SUSY SM (MSSM) [34] within factor 3 by choosing these parameters. Once we find the parameter set at the GUT scale that realizes the observed quark and lepton masses and mixings at the low-energy scale in an explicit model, then we can predict various partial decay widths of the nucleon.

In this paper, we calculate the various decay widths of the nucleons in the  $E_6 \times SU(2)_F \times U(1)_A$  SUSY GUTs. If the parameter sets, which realize the observed quark and lepton masses and mixings, have been found easily, we can calculate the various decay widths by the parameter sets. However, it is not an easy task to find the parameter sets in calculating renormalization group equations (RGEs) that are dependent on the explicit GUT models. Alternatively, we find the relations between diagonalizing matrices that are independent of the renormalization scale, and under the relations we calculate the various nucleon decay widths. Moreover, we re-examine the conditions for the identification of the grand unification group by using 100–1000 times more model points than in the previous paper.

## 2. $E_6 \times SU(2)_F \times U(1)_A$ SUSY GUT model

In this section we introduce the  $E_6 \times SU(2)_F \times U(1)_A$  SUSY GUT model [24–31], and the diagonalizing matrices in the model are derived. The setting and notation for the model in this paper are basically the same as those for the model in Ref. [31]. The diagonalizing matrix of light neutrinos,  $L_\nu$ , is derived from the Maki–Nakagawa–Sakata (MNS) matrix [35] and the diagonalizing matrix of charged leptons,  $L_e$ , through the relation on the MNS matrix as  $U_{\text{MNS}} = L_\nu^\dagger L_e$ . Therefore, we omit the explanation for the derivation of neutrino mass matrices in the model. It is shown in Ref. [31] in detail.

One of the most important features of the anomalous  $U(1)_A$  gauge theory is that the VEVs of the GUT singlet operators  $O_i$  are determined by their  $U(1)_A$  charges  $o_i$  as

$$\langle O_i \rangle = \begin{cases} 0 & (o_i > 0) \\ \lambda^{-o_i} & (o_i \leq 0) \end{cases}, \quad (2)$$

where  $\lambda$  is determined from the FI parameter  $\xi$  as  $\lambda \equiv \xi/\Lambda$ . In this paper, we take  $\lambda \sim 0.22$  and the notation in which the cutoff  $\Lambda = 1$ . As a result, the coefficient of the term  $XYZ$  is determined by their  $U(1)_A$  charges,  $x$ ,  $y$ , and  $z$  as  $\lambda^{x+y+z} XYZ$  if  $x + y + z \geq 0$ , and they vanish if  $x + y + z < 0$ . These features are important in understanding the following arguments in this paper.

The contents of matters and Higgs and their charge assignments are shown in Table 1. In this paper, the capital letter denotes the superfield and the small letter denotes the corresponding  $U(1)_A$  charge. The 27 dimensional (fundamental) representation of the  $E_6$  group, **27**, is decomposed in the  $E_6 \supset SO(10) \times U(1)_{V'}$  notation (and  $[SO(10) \supset SU(5) \times U(1)_V]$  notation) as

$$\mathbf{27} = \mathbf{16}_1[\mathbf{10}_1 + \bar{\mathbf{5}}_{-3} + \mathbf{1}_5] + \mathbf{10}_{-2}[\mathbf{5}_{-2} + \bar{\mathbf{5}}'_2] + \mathbf{1}'_4[\mathbf{1}'_0]. \quad (3)$$

**16** and **10** of  $SO(10)$  are decomposed in the  $SU(3)_C \times SU(2)_L \times U(1)_Y$  notation as

$$\mathbf{16} \rightarrow \underbrace{q_L(\mathbf{3}, \mathbf{2})_{\frac{1}{6}} + u_R^c(\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}} + e_R^c(\mathbf{1}, \mathbf{1})_1}_{\mathbf{10}} + \underbrace{d_R^c(\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} + l_L(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}}_{\bar{\mathbf{5}}} + \underbrace{v_R^c(\mathbf{1}, \mathbf{1})_0}_{\mathbf{1}}, \quad (4)$$

$$\mathbf{10} \rightarrow \underbrace{D_R^c(\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} + L_L(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}}_{\bar{\mathbf{5}}'} + \underbrace{\bar{D}_R^c(\mathbf{3}, \mathbf{1})_{-\frac{1}{3}} + \bar{L}_L(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}}_{\mathbf{5}}, \quad \mathbf{1}' \rightarrow \underbrace{N_R^c(\mathbf{1}, \mathbf{1})_0}_{\mathbf{1}}. \quad (5)$$

**27** includes two  $\bar{\mathbf{5}}$ s and two  $\mathbf{1}$ s. This feature plays an important role in realizing realistic quark and lepton masses and mixings in this model.  $F_a$  and  $\bar{F}^a$  are Higgs that obtain VEVs  $\langle F_a \rangle$  and  $\langle \bar{F}^a \rangle$  as

$$\langle F_a \rangle \sim \begin{pmatrix} 0 \\ e^{i\rho} \lambda^{-(f+\bar{f})/2} \end{pmatrix}, \quad \langle \bar{F}^a \rangle \sim \begin{pmatrix} 0 \\ \lambda^{-(f+\bar{f})/2} \end{pmatrix}, \quad (6)$$

and break  $SU(2)_F$ .  $\Phi$  and  $\bar{\Phi}$  are Higgs that obtain VEVs  $\langle \Phi \rangle$  and  $\langle \bar{\Phi} \rangle$  in the  $SO(10)$  singlet direction as  $\langle \mathbf{1}'_\Phi \rangle = \langle \mathbf{1}'_{\bar{\Phi}} \rangle = v_\phi \sim \lambda^{-(\phi+\bar{\phi})/2}$  and break  $E_6$  into  $SO(10)$ .  $C$  and  $\bar{C}$  are Higgs that obtain VEVs  $\langle C \rangle$  and  $\langle \bar{C} \rangle$  in the  $SU(5)$  singlet direction as  $\langle \mathbf{16}_C \rangle = \langle \bar{\mathbf{16}}_{\bar{C}} \rangle = v_c \sim \lambda^{-(c+\bar{c})/2}$  and break  $SO(10)$  into  $SU(5)$ .  $A$  is an adjoint Higgs that is decomposed in  $SO(10) \times U(1)_{V'}$  notation as  $\mathbf{78} \rightarrow \mathbf{45}_0 + \mathbf{16}_{-3} + \bar{\mathbf{16}}_3 + \mathbf{1}_0$ .  $A$  obtains a Dimopoulos–Wilczek-type VEV  $\langle A \rangle$  [36,37] as

$$\langle \mathbf{45}_A \rangle = i\sigma_2 \times \begin{pmatrix} x & & & \\ & x & & \\ & & x & \\ & & & 0 \\ & & & & 0 \end{pmatrix}, \quad (7)$$

**Table 1.** Field contents and charge assignments under  $E_6 \times SU(2)_F \times U(1)_A \times Z_6$ .

|           | $\Psi_a$  | $\Psi_3$      | $F_a$          | $\bar{F}^a$                 | $\Phi$    | $\bar{\Phi}$                 | $C$       | $\bar{C}$                    | $A$            | $Z_3$          | $\Theta$ |
|-----------|-----------|---------------|----------------|-----------------------------|-----------|------------------------------|-----------|------------------------------|----------------|----------------|----------|
| $E_6$     | <b>27</b> | <b>27</b>     | <b>1</b>       | <b>1</b>                    | <b>27</b> | <b><math>\bar{27}</math></b> | <b>27</b> | <b><math>\bar{27}</math></b> | <b>78</b>      | <b>1</b>       | <b>1</b> |
| $SU(2)_F$ | <b>2</b>  | <b>1</b>      | <b>2</b>       | <b><math>\bar{2}</math></b> | <b>1</b>  | <b>1</b>                     | <b>1</b>  | <b>1</b>                     | <b>1</b>       | <b>1</b>       | <b>1</b> |
| $U(1)_A$  | 4         | $\frac{3}{2}$ | $-\frac{3}{2}$ | $-\frac{5}{2}$              | -3        | 1                            | -4        | -1                           | $-\frac{1}{2}$ | $-\frac{3}{2}$ | -1       |
| $Z_6$     | 3         | 3             | 1              | 0                           | 0         | 0                            | 5         | 0                            | 0              | 0              | 0        |

which is proportional to the  $B - L$  charge. This VEV of the adjoint Higgs plays an important role in realizing the doublet–triplet splitting. Here,  $\sigma_i$  ( $i = 1, 2, 3$ ) are the Pauli matrices. Note that these VEVs of all fields except  $\langle F \rangle$  are real. Such a situation can be realized in a concrete model [28–31].

This model includes the MSSM doublet Higgs  $H_u$  and  $H_d$  as [28,29]

$$H_u \subset \mathbf{5}_\Phi, \quad (8)$$

$$H_d \subset \bar{\mathbf{5}}'_\Phi + \beta_H e^{i\delta} \lambda^{0.5} \bar{\mathbf{5}}_C, \quad (9)$$

where  $\beta_H$  is a real  $O(1)$  coefficient.  $\delta$  is a complex phase and depends on the models. Please see Refs. [17–19,21] to understand how to realize the doublet–triplet splitting in a natural way. Yukawa couplings are derived from the superpotential,

$$W_Y = (a\Psi_3\Psi_3 + b\Psi_3\bar{F}^a\Psi_a + c\bar{F}^a\Psi_a\bar{F}^b\Psi_b)\Phi + d(\Psi_a, \Phi, \bar{\Phi}, A, Z_3, \Theta) \\ + f'\bar{F}^a\Psi_a\epsilon^{bc}F_b\Psi_cC + g'\Psi_3\epsilon^{ab}F_a\Psi_bC, \quad (10)$$

where  $a, b, c, f'$ , and  $g'$  are  $O(1)$  coefficients, and  $d(\Psi_a, \Phi, \bar{\Phi}, A, Z_3, \Theta)$  is a gauge-invariant function of  $\Psi_a, \Phi, \bar{\Phi}, A, Z_3$ , and  $\Theta$ , which contributes to  $\Psi_1\Psi_2\Phi$ . Note that the operator  $\epsilon^{ab}\Psi_a\Psi_b\Phi$  is not allowed because of the antisymmetric feature of  $\epsilon^{ab}$ , where  $\epsilon^{12} = -\epsilon^{21} = 1$  is an antisymmetric tensor of the  $SU(2)_F$  group. Therefore, the function  $d$  includes, e.g.,  $\Theta^3 Z_3 \epsilon^{ab} \Psi_a A \Psi_b \Phi$ ,  $\Theta^3 (\bar{\Phi} \epsilon^{ab} \Psi_a) (\Psi_b \Phi \Phi)$ ,  $\dots$ . This function contains the following terms by developing the VEVs of  $A, \Phi, \bar{\Phi}, Z_3$ , and  $\Theta$ :

$$d(\Psi_a, \Phi, \bar{\Phi}, A, Z_3, \Theta) \rightarrow \frac{2}{3} d_5 \lambda^5 \epsilon^{ab} D_{R\Psi_a}^c \bar{D}_{R\Psi_b}^c (N_R^c)_\Phi, \quad \frac{1}{3} d_q \lambda^5 \epsilon^{ab} q_{L\Psi_a} u_{R\Psi_b}^c (\bar{L}_L)_\Phi, \\ \frac{1}{3} d_q \lambda^5 \epsilon^{ab} q_{L\Psi_a} d_{R\Psi_b}^c (L_L)_\Phi, \quad -d_l \lambda^5 \epsilon^{ab} l_{L\Psi_a} e_{R\Psi_b}^c (L_L)_\Phi, \quad -d_l \lambda^5 \epsilon^{ab} l_{L\Psi_a} \nu_{R\Psi_b}^c (\bar{L}_L)_\Phi, \\ h \lambda^5 \epsilon^{ab} L_{L\Psi_a} N_{R\Psi_b}^c (\bar{L}_L)_\Phi, \quad (11)$$

where  $d_5, d_q, d_l$ , and  $h$  are real  $O(1)$  coefficients. Note that the coefficients of the first 5 terms in Eq. (11) are proportional to the  $B - L$  charge. The reason is as follows. The above argument on the antisymmetric feature can be applied to the terms  $\epsilon^{ab} \mathbf{16}_a \mathbf{16}_b \mathbf{10}_\Phi$  and  $\epsilon^{ab} \mathbf{10}_a \mathbf{10}_b \mathbf{1}_\Phi$  of  $SO(10)$ . To obtain non-zero terms, they must pick up the breaking of  $SO(10)$ , i.e., the adjoint Higgs VEV  $\langle A \rangle$  that is proportional to the  $B - L$  charge. Since there are several terms whose contributions are not proportional to the  $B - L$  charge, e.g.,  $\Theta^2 Z_3 \epsilon^{ab} \Psi_a A^3 \Psi_b \Phi$ , we introduce different  $O(1)$  parameters,  $d_5, d_q$ , and  $d_l$  (see Appendix B).

The up-type quark Yukawa matrix  $Y_u$  is derived as

$$Y_u = \begin{pmatrix} 0 & \frac{1}{3} d_q \lambda^5 & 0 \\ -\frac{1}{3} d_q \lambda^5 & c \lambda^4 & b \lambda^2 \\ 0 & b \lambda^2 & a \end{pmatrix} \equiv \begin{pmatrix} 0 & \frac{1}{3} y_{u12} \lambda^5 & 0 \\ -\frac{1}{3} y_{u12} \lambda^5 & y_{u22} \lambda^4 & y_{u23} \lambda^2 \\ 0 & y_{u23} \lambda^2 & y_{u33} \end{pmatrix}, \quad (12)$$

where  $y_{uij}$  ( $i, j = 1, 2, 3$ ) is a real  $O(1)$  coefficient. The factor  $\frac{1}{3}$  in  $(Y_u)_{12}$  and  $(Y_u)_{21}$  plays an important role in obtaining a small up quark mass.

Next, we derive down-type quark and charged lepton Yukawa matrices. Note that three **27** matters of  $E_6$  include six  $\bar{\mathbf{5}}\text{s}$  of  $SU(5)$ . Three of the six  $\bar{\mathbf{5}}\text{s}$  become superheavy with three  $\mathbf{5}\text{s}$  after developing the VEVs of  $\Phi$  and  $C$ . The other three  $\bar{\mathbf{5}}\text{s}$  are massless, corresponding to the SM  $\bar{\mathbf{5}}\text{s}$ . To obtain the SM  $\bar{\mathbf{5}}\text{s}$ , we estimate the mass matrix for  $\mathbf{5}$  and  $\bar{\mathbf{5}}$ . Take the relation

$$\frac{\lambda^c \langle C \rangle}{\lambda^\phi \langle \Phi \rangle} = r \lambda^{0.5}, \quad (13)$$

which is important in obtaining realistic large neutrino mixings. Here  $r$  is a real  $O(1)$  coefficient. Then, the mass matrix for  $\mathbf{5}$  and  $\bar{\mathbf{5}}$  is derived as

$$\left( \begin{array}{ccc|ccc} 0 & \alpha d_5 \lambda^5 & 0 & 0 & f e^{i\rho} \lambda^{5.5} & g e^{i\rho} \lambda^{3.5} \\ -\alpha d_5 \lambda^5 & c \lambda^4 & b \lambda^2 & f e^{i\rho} \lambda^{5.5} & 0 & 0 \\ 0 & b \lambda^2 & a & g e^{i\rho} \lambda^{3.5} & 0 & 0 \end{array} \right) \equiv (M_1 \mid M_2), \quad (14)$$

where we redefine the real  $O(1)$  parameters  $f'$  and  $g'$  as  $f \equiv r f'$  and  $g \equiv r g'$ . Here,  $\alpha = 2/3$  for the triplet (colored) component and  $\alpha = 0$  for the doublet component. We diagonalize the  $3 \times 6$  mass matrix  $(M_1 \mid M_2)$  as

$$V^\dagger (M_1 \mid M_2) U = (M_H^{\text{diag}} \mid 0), \quad (15)$$

where  $V$  is a  $3 \times 3$  unitary matrix and  $U$  is a  $6 \times 6$  unitary matrix, which is given as

$$U \equiv \begin{pmatrix} U_{10}^H & U_{10}^0 \\ U_{16}^H & U_{16}^0 \end{pmatrix}. \quad (16)$$

The massless  $\bar{\mathbf{5}}_i^0$  are given as

$$\bar{\mathbf{5}}_i^0 \equiv (U_{10}^{0\dagger})_{ij} \bar{\mathbf{5}}'_j + (U_{16}^{0\dagger})_{ij} \bar{\mathbf{5}}_j = \begin{pmatrix} \bar{\mathbf{5}}_1 + \cdots \\ \bar{\mathbf{5}}'_1 + \cdots \\ \bar{\mathbf{5}}_2 + \cdots \end{pmatrix}, \quad (17)$$

where  $U_{10}^0$  and  $U_{16}^0$  are calculated as

$$U_{10}^0 = \begin{pmatrix} -\frac{a\alpha d_5(bg - af)}{(ac - b^2)^2} \lambda^{2.5} e^{i\rho} & 1 & \mathcal{O}(\lambda^{5.5}) \\ \frac{bg - af}{ac - b^2} \lambda^{1.5} e^{i\rho} & \frac{a\alpha d_5}{ac - b^2} \lambda & \mathcal{O}(\lambda^{4.5}) \\ -\left(\frac{g}{a} + \frac{b}{a} \frac{bg - af}{ac - b^2}\right) \lambda^{3.5} e^{i\rho} & -\frac{b\alpha d_5}{ac - b^2} \lambda^3 & \mathcal{O}(\lambda^{6.5}) \end{pmatrix}, \quad (18)$$

$$U_{16}^0 = \begin{pmatrix} 1 & 0 & 0 \\ \mathcal{O}(\lambda^6) & 0 & 1 \\ -\frac{bg - af}{ac - b^2} \frac{\alpha d_5}{g} \lambda^3 & -\frac{\alpha d_5^2}{ac - b^2} \frac{a}{g} \lambda^{2.5} e^{-i\rho} & -\frac{f}{g} \lambda^2 \end{pmatrix}. \quad (19)$$

The detailed derivation is shown in Refs. [30,31].

As a result, the down-type Yukawa matrix  $Y_d$  is given as

$$\begin{aligned}
 Y_d &= Y^\Phi U_{16}^0 + \beta_H e^{-i\delta} \lambda^{0.5} Y^C U_{10}^0 \\
 &= \begin{pmatrix} \left[ \frac{bg - af}{ac - b^2} \left( f' - \frac{bg'}{a} \right) - \frac{gg'}{a} \right] \beta_H e^{i(2\rho - \delta)} \lambda^6 & & \\ & \left( -\frac{d_q}{3} - \frac{bg - af}{ac - b^2} \frac{b^2 d_5}{g} \right) \lambda^5 & \\ & -\frac{bg - af}{ac - b^2} \frac{a^2 d_5}{g} \lambda^3 & \\ & -\frac{bg' - af'}{ac - b^2} \frac{2}{3} d_5 \beta_H e^{i(\rho - \delta)} \lambda^{5.5} & \frac{1}{3} d_q \lambda^5 \\ & \left( -\frac{(\frac{2}{3} d_5)^2}{ac - b^2} \frac{ab}{g} e^{-i\rho} + f' \beta_H e^{i(\rho - \delta)} \right) \lambda^{4.5} & \left( \frac{ac - b^2}{a} + \frac{bg - af}{g} \frac{b}{a} \right) \lambda^4 \\ & \left( -\frac{(\frac{2}{3} d_5)^2}{ac - b^2} \frac{a^2}{g} e^{-i\rho} + g' \beta_H e^{i(\rho - \delta)} \right) \lambda^{2.5} & \frac{bg - af}{g} \lambda^2 \end{pmatrix} \\
 &\equiv \begin{pmatrix} y_{d11} \lambda^6 & \frac{2}{3} y_{d12} \lambda^{5.5} & \frac{1}{3} y_{d13} \lambda^5 \\ \frac{2}{3} y_{d21} \lambda^5 & y_{d22} \lambda^{4.5} & y_{d23} \lambda^4 \\ \frac{2}{3} y_{d31} \lambda^3 & y_{d32} \lambda^{2.5} & y_{d33} \lambda^2 \end{pmatrix}, \tag{20}
 \end{aligned}$$

where  $y_{dij}$  is a  $O(1)$  coefficient that includes complex phase. In our calculation for nucleon decay,  $y_{dij}$  is taken to be a real  $O(1)$  coefficient for simplicity. Here,

$$Y^\Phi = \begin{pmatrix} 0 & \frac{1}{3} d_q \lambda^5 & 0 \\ -\frac{1}{3} d_q \lambda^5 & c \lambda^4 & b \lambda^2 \\ 0 & b \lambda^2 & a \end{pmatrix}, \quad Y^C = \begin{pmatrix} 0 & f' e^{i\rho} \lambda^4 & g' e^{i\rho} \lambda^2 \\ f' e^{i\rho} \lambda^4 & 0 & 0 \\ g' e^{i\rho} \lambda^2 & 0 & 0 \end{pmatrix}. \tag{21}$$

The charged lepton Yukawa matrix  $Y_e$  is derived from a relationship  $Y_e = Y_d^T$  with  $\alpha = 0$  ( $d_5 = 0$ ) and  $d_q/3 \rightarrow -d_l$  as

$$\begin{aligned}
 Y_e &= \begin{pmatrix} \left[ \frac{bg - af}{ac - b^2} \left( f' - \frac{bg'}{a} \right) - \frac{gg'}{a} \right] \beta_H e^{i(2\rho - \delta)} \lambda^6 & d_l \lambda^5 & 0 \\ 0 & f' \beta_H e^{i(\rho - \delta)} \lambda^{4.5} & g' \beta_H e^{i(\rho - \delta)} \lambda^{2.5} \\ -d_l \lambda^5 & \left( \frac{ac - b^2}{a} + \frac{bg - af}{g} \frac{b}{a} \right) \lambda^4 & \frac{bg - af}{g} \lambda^2 \end{pmatrix} \\
 &\equiv \begin{pmatrix} y_{d11} \lambda^6 & y_{e12} \lambda^5 & 0 \\ 0 & y_{e22} \lambda^{4.5} & y_{e23} \lambda^{2.5} \\ -y_{e12} \lambda^5 & y_{d23} \lambda^4 & y_{d33} \lambda^2 \end{pmatrix} \tag{22}
 \end{aligned}$$

where  $y_{eij}$  is an  $O(1)$  coefficient that includes complex phase. Again, we take  $y_{eij}$  as a real  $O(1)$  coefficient for simplicity. Finally, to obtain  $Y_u$ ,  $Y_d$ , and  $Y_e$  we use 16 real parameters,  $y_{uij}$ ,  $y_{dij}$ , and  $y_{eij}$ . In the original  $E_6 \times SU(2)_F \times U(1)_A$  models, we have 9 real parameters and 2 CP phases. Therefore, we have several relations among  $y_{uij}$ ,  $y_{dij}$ , and  $y_{eij}$ . We will discuss these relations in the next section.



Let us diagonalize these Yukawa matrices by field redefinition as

$$\begin{aligned}\psi_{Li} Y_{ij} \psi_{Rj}^c &= (L_\psi^\dagger \psi_L)_i (L_\psi^T Y R_\psi)_{ij} (R_\psi^\dagger \psi_R^c)_j \\ &= \psi_{Li}' Y_{\text{diag } ij} \psi_{Rj}'^c,\end{aligned}\quad (23)$$

where  $\psi$  is a gauge eigenstate field and  $\psi'$  is a mass eigenstate field. We summarize the detailed calculation in Appendix A. The diagonalizing matrices are calculated as

$$L_u \sim \begin{pmatrix} 1 & \frac{1}{3}\lambda & 0 \\ \frac{1}{3}\lambda & 1 & \lambda^2 \\ \frac{1}{3}\lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad R_u \sim \begin{pmatrix} 1 & \frac{1}{3}\lambda & 0 \\ \frac{1}{3}\lambda & 1 & \lambda^2 \\ \frac{1}{3}\lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad (24)$$

$$L_d \sim \begin{pmatrix} 1 & \frac{2}{3}\lambda & \frac{1}{3}\lambda^3 \\ \frac{2}{3}\lambda & 1 & \lambda^2 \\ \frac{2}{3}\lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad R_d \sim \begin{pmatrix} 1 & \frac{2}{3}\lambda^{0.5} & \frac{2}{3}\lambda \\ \frac{2}{3}\lambda^{0.5} & 1 & \lambda^{0.5} \\ \frac{2}{3}\lambda & \lambda^{0.5} & 1 \end{pmatrix}, \quad (25)$$

$$L_e \sim \begin{pmatrix} 1 & \lambda^{0.5} & 0 \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}, \quad R_e \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad (26)$$

$$L_\nu \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}. \quad (27)$$

Since this model has a lot of  $O(1)$  parameters for the right-handed neutrino mass matrix, we do not have any interesting relations in  $L_\nu$ . Realistic CKM and MNS matrices can be obtained as

$$U_{\text{CKM}} = L_u^\dagger L_d \sim \begin{pmatrix} 1 & \frac{2}{3}\lambda & \lambda^4 \\ \frac{2}{3}\lambda & 1 & \lambda^2 \\ \frac{2}{3}\lambda^3 & \lambda^2 & 1 \end{pmatrix}, \quad U_{\text{MNS}} = L_\nu^\dagger L_e \sim \begin{pmatrix} 1 & \lambda^{0.5} & \lambda \\ \lambda^{0.5} & 1 & \lambda^{0.5} \\ \lambda & \lambda^{0.5} & 1 \end{pmatrix}, \quad (28)$$

if we consider the  $O(1)$  coefficients. Since the coefficient of  $(U_{\text{CKM}})_{13}$  is vanishing in leading order in this model [30], the sub-leading contribution  $\lambda^4$  is dominant. As noted previously, we estimate  $L_\nu$  from the observed  $U_{\text{MNS}}$  and  $L_e$ .

### 3. Conditions for the diagonalizing matrices

In the original  $E_6 \times SU(2)_F \times U(1)_A$  SUSY GUT models with spontaneously broken CP symmetry, the number of parameters for the Yukawa couplings of up quarks, down quarks, and charged leptons is 9 (real parameters) + 2 (CP phases), which is smaller than the number of observed parameters of masses and mixings. Therefore, once we fix these parameters from the observed values of masses and mixings, we can predict all diagonalizing matrices. The main obstacle for this approach is that these Yukawa couplings are determined at the GUT scale. If the masses and mixings at the GUT scale had been calculated from these measured parameters through the renormalization group equations (RGEs), we would adopt this approach. Unfortunately, many new couplings, which can contribute to the running of the Yukawa couplings, appear, when superheavy fields appear at the mass scales that are dependent on the models. Of course, once we fix the GUT models, we can calculate the low-energy effective theory. However, it is not an easy task to fix the 11 parameters to satisfy the measured quark and lepton masses and mixings by RGEs that change when superheavy fields decouple, though we can do it in principle.



Therefore, in this paper, we adopt another approach. We select several relations between Yukawa couplings that are not strongly dependent on the renormalization scale. Using these relations, we reduce the number of parameters.

As noted in the previous section, we consider real Yukawa couplings for simplicity. Then, generically, we have 27 parameters for the Yukawa couplings of up quarks, down quarks, and charged leptons. In the previous section, we introduced 16 real parameters  $y_{uij}$ ,  $y_{dij}$ , and  $y_{eij}$  for these Yukawa couplings. Therefore, there must be 11 ( $= 27 - 16$ ) relations among the parameters of masses and diagonalizing matrices. In the following, the notation of the angles is defined in Appendix A. From  $(Y_u)_{13} = (Y_u)_{31} = (Y_e)_{13} = 0$ , the relations

$$s_{13}^{uL} = 0, s_{13}^{uR} = 0, s_{13}^{eL} = 0 \quad (29)$$

are derived, respectively.  $(Y_u)_{23} = (Y_u)_{32}$ ,  $(Y_u)_{12} = -(Y_u)_{21}$ , and  $(Y_u)_{11} = 0$  result in

$$s_{23}^{uL} = s_{23}^{uR}, s_{12}^{uL} = -s_{12}^{uR}, (s_{12}^{uL})^2 = m_u/m_c. \quad (30)$$

$(Y_e)_{31} = -(Y_e)_{12}$  and  $(Y_e)_{21} = 0$  lead to

$$s_{12}^{eR} = s_{23}^{eL} s_{12}^{eL}, s_{13}^{eR} = -s_{12}^{eL} m_\mu/m_\tau. \quad (31)$$

From the relations  $(Y_d)_{33} = (Y_e)_{33}$  and  $(Y_d)_{23} = (Y_e)_{32}$ ,

$$s_{23}^{dL} = s_{23}^{eR}, m_b = m_\tau \quad (32)$$

are derived. The relation  $m_b = m_\tau$  is useless for fixing the diagonalizing matrices. Finally, when we consider the relation  $(Y_e)_{11} = (Y_d)_{11}$  in addition to the above relations, we obtain

$$s_{13}^{eL} s_{13}^{eR} m_\tau + s_{12}^{eL} s_{12}^{eR} m_\mu + m_e = s_{13}^{dL} s_{13}^{dR} m_b + s_{12}^{dL} s_{12}^{dR} m_s + m_d. \quad (33)$$

In our analysis, we do not use the last relation because it is strongly dependent on the renormalization scale. As a result, we use 9 relations in our analysis. We have checked the scale dependence of these relations by explicit numerical calculations of the RGEs in the MSSM [38,39].

We have an additional 7 ( $= 16 - 9$ ) relations because the original models have only 9 real parameters ( $a$ ,  $b$ ,  $c$ ,  $d_q$ ,  $d_5$ ,  $d_l$ ,  $f$ ,  $g$ , and  $\beta_H$ ) if we take vanishing CP phases. Unfortunately, these are strongly dependent on the renormalization scale, and therefore, we do not use these relations in our analysis.

As a result, we use only 6 parameters in our numerical calculations of nucleon decays for 7 diagonalizing matrices  $L_u$ ,  $L_d$ ,  $L_e$ ,  $L_\nu$ ,  $R_u$ ,  $R_d$ ,  $R_e$ . Since we assume real diagonalizing matrices, each matrix has three real parameters, generically. The CKM matrix and the MNS matrix reduce the 21 parameters to 15 parameters, and, because of the 9 relations, only 6 ( $= 15 - 9$ ) parameters are sufficient. (Strictly speaking, the signature of  $s_{12}^{uL}$  is an additional parameter because the relation  $(s_{12}^{uL})^2 = m_u/m_c$  cannot fix the signature.) Note that we have used 12 parameters for fixing the real diagonalizing matrices of the  $E_6 \times U(1)_A$  GUT models in the previous paper. In this paper, we have succeeded in reducing the number of parameters by half.

#### 4. Numerical calculation

In this section, we calculate various partial decay widths of nucleons numerically, and compare the results with those in the previous paper [1].

In our calculation, we use the VEVs

$$x = 1 \times 10^{16} \text{ GeV}, \quad v_c = 5 \times 10^{14} \text{ GeV}, \quad v_\phi = 5 \times 10^{15} \text{ GeV}, \quad (34)$$

where  $x$  is the scale of the adjoint Higgs VEV that breaks  $E_6$  into  $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times U(1)_{V'}$ ,  $v_\phi$  is the VEV of  $\Phi$  that breaks  $U(1)_{V'}$ , and  $v_c$  is the Higgs VEV that breaks  $SU(2)_R \times U(1)_{B-L}$  into  $U(1)_Y$ . These VEVs are the same as the VEVs of the GUT Higgs adopted in the previous paper<sup>1</sup>. The  $X$ -type gauge boson masses are written as

$$M_X = g_{\text{GUT}} x, \quad M_{X'} = g_{\text{GUT}} \sqrt{x^2 + v_c^2}, \quad M_{X''} = g_{\text{GUT}} \sqrt{\frac{x^2}{4} + v_\phi^2}. \quad (35)$$

Since the larger  $x$  leads to larger  $M_X/M_{X'}$  and  $M_X/M_{X''}$ , the differences between the different unification groups become clearer in the nucleon decay processes for larger  $x$ .

We generate the real diagonalizing matrices  $L_u, L_d, L_e, L_\nu, R_u, R_d$ , and  $R_e$  as follows.

1. Once  $\theta_{23}^{uL}$  ( $s_{23}^{uL} = \sin \theta_{23}^{uL}$ ) is generated through the relation  $\theta_{23}^{uL} = B_{23}^{uL} \lambda^2$ , where  $B_{23}^{uL}$  is the  $O(1)$  coefficient determined randomly from 0.5 to 2,  $L_u$  and  $R_u$  can be fixed by the relations (29) and (30).
2. The three parameters for  $R_d$  are generated randomly in the same manner as  $\theta_{23}^{uL}$ .  $L_d$  can be determined by  $L_d = L_u U_{\text{CKM}}^{(\text{exp})}$ .
3. Once we generate two parameters among six for  $L_e$  and  $R_e$  randomly, we can fix the other 4 parameters by the relations (29), (31), and (32).
4.  $L_\nu$  can be determined by  $L_\nu = L_e U_{\text{MNS}}^{(\text{exp})}$ .
5. We check whether all  $O(1)$  coefficients  $B_{ij}$  of the components of the diagonalizing matrices ( $L_u, L_d, L_e, L_\nu, R_u, R_d$ , and  $R_e$ ) in Eqs. (24)–(27) are within the region  $0.5 \leq B_{ij} \leq 2$  or not. We adopt the parameter set only if all coefficients satisfy the condition.

In the above calculation we use  $m_u/m_c = 0.0021$ ,  $m_\mu/m_\tau = 0.059$  [40–44]:

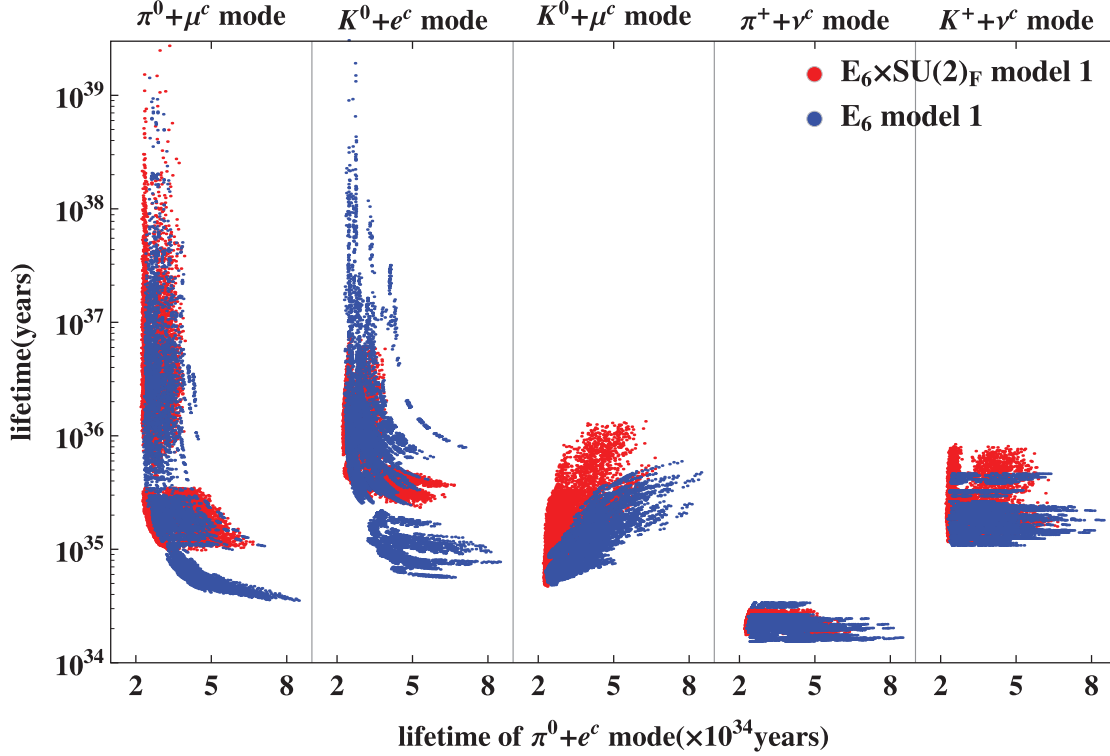
$$U_{\text{CKM}}^{(\text{exp})} = \begin{pmatrix} 0.97 & 0.23 & 0.0035 \\ -0.23 & 0.97 & 0.041 \\ 0.0086 & -0.040 & 1.0 \end{pmatrix}, \quad U_{\text{MNS}}^{(\text{exp})} = \begin{pmatrix} 0.83 & 0.54 & 0.15 \\ -0.48 & 0.53 & 0.70 \\ 0.30 & -0.65 & 0.70 \end{pmatrix}. \quad (36)$$

Although there are errors in these quark and lepton mixings, we do not consider the errors in our numerical calculation. Following the above procedure, we have generated  $10^4$ – $10^6$  model points and calculated various partial decay widths of nucleons.

#### 4.1. Various decay modes for the proton

We calculate the proton lifetimes for various decay modes in the  $E_6 \times SU(2)_F$  model. Basically, we follow the procedure in the previous paper [1]. Namely, we use the hadron matrix elements calculated by QCD lattice [45], and use the renormalization factor of the minimal SUSY  $SU(5)$  GUT as  $A_R = 3.6$  for the dimension-6 operators that include a right-handed charged lepton  $e_R^c$ , and  $A_R = 3.4$  for the operators that include a doublet lepton  $l_L$  as the reference value [46]. In order to apply our results to an explicit GUT model, a correction for the renormalization factor is required. (For example, the renormalization factors of the  $E_6$  GUT model defined by Table II in Ref. [21] become  $A_R = 3.6$  for the operators with  $e_R^c$  and  $A_R = 3.5$  for the operators with  $l_L$ . As noted in the previous paper [1],

<sup>1</sup> These VEVs correspond to Choices A and B in Appendix B.



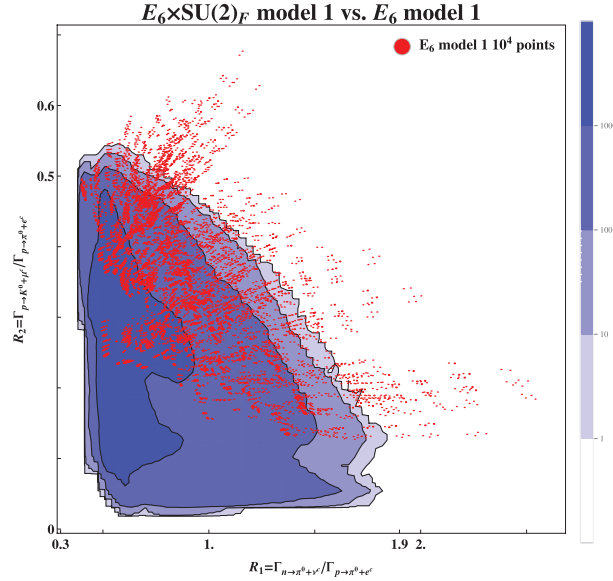
**Fig. 1.** The proton lifetimes for various decay modes in the  $E_6 \times SU(2)_F$  model 1 and the  $E_6$  model 1 with  $M_X = g_{\text{GUT}}x$ ,  $M_{X'} = g_{\text{GUT}}\sqrt{x^2 + v_c^2}$ ,  $M_{X''} = g_{\text{GUT}}\sqrt{\frac{x^2}{4} + v_\phi^2}$ ,  $x = 1 \times 10^{16}$  GeV,  $v_c = 5 \times 10^{14}$  GeV,  $v_\phi = 5 \times 10^{15}$  GeV. Each model has  $10^4$  model points.

larger gauge couplings caused by a lot of superheavy fields in this model make the renormalization factor larger, while a smaller unification scale makes the renormalization factor smaller. As a result, the renormalization factor in this model is not so different from the factor in the minimal SUSY  $SU(5)$ .) The results are shown in Fig. 1. In the figure, we take the partial lifetime of  $p \rightarrow \pi^0 + e^c$  as the horizontal axis and the partial lifetime of the other decay modes as the vertical axis. In Fig. 1, we show the predictions of the  $E_6$  model in the previous paper as well as those of  $E_6 \times SU(2)_F$  model.

We have two comments on these results. First, in many model points of the  $E_6 \times SU(2)_F$  model the lifetime of the  $p \rightarrow \pi^0 + e^c$  mode is shorter than the lifetimes in the  $E_6$  model. This result comes from larger  $(L_e)_{11}$ , because  $s_{12}^{eL}$  is smaller and  $s_{13}^{eL}$  is vanishing, as in (29). The smaller  $s_{12}^{eL}$  is caused by the last relation in (31),  $m_\mu/m_\tau > \lambda^2$  and  $s_{13}^{eR} \sim \lambda^3$ . On the other hand, the lifetime of the  $p \rightarrow K^0 + \mu^c$  mode does not become short because  $(L_e)_{22}$  does not become larger. This is because the  $s_{23}^{eL}$  has a large value because of the first relation in (31), though  $s_{12}^{eL}$  is smaller. As a result of the two opposite effects from smaller  $s_{12}^{eL}$  and larger  $s_{23}^{eL}$ , the prediction of the  $p \rightarrow K^0 + \mu^c$  mode becomes wider than in the  $E_6$  model. Second, the lifetimes for the  $K^0 + e^c$  and  $\pi^0 + \mu^c$  modes become longer, as can be seen in Fig. 1. This is also because of the smaller  $s_{12}^{eL}$ .

#### 4.2. Calculation of $R_1$ and $R_2$ in $E_6 \times SU(2)_F \times U(1)_A$

In the previous paper [1], we emphasized that the parameters  $R_1 = \frac{\Gamma_{p \rightarrow \pi^0 + \nu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}}$  and  $R_2 = \frac{\Gamma_{p \rightarrow K^0 + \mu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}}$  are useful to identify the grand unification groups,  $SU(5)$ ,  $SO(10)$ , or  $E_6$ , in the anomalous  $U(1)_A$

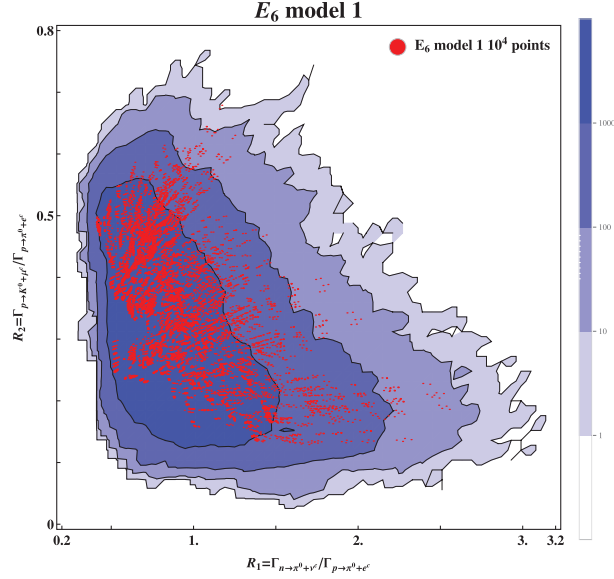


**Fig. 2.** Contour plot of  $E_6 \times SU(2)_F$  model point density. The model point density is defined by the number of model points per unit area  $(\Delta R_1, \Delta R_2) = ((1.9 - 0.3)/50 = 0.032, 0.6/50 = 0.012)$  in the  $(R_1, R_2)$  plain after generating  $10^6$  model points.  $10^4$  model points for the  $E_6$  model without  $SU(2)_F$ , which are calculated in Ref. [1], are dotted. VEVs are taken as  $x = 1 \times 10^{16}$  GeV,  $v_c = 5 \times 10^{14}$  GeV, and  $v_\phi = 5 \times 10^{15}$  GeV.

GUTs.  $R_1$  can be important to identify the size of the rank of the unification group [22,23].  $R_2$  is sensitive to the Yukawa structure, especially for second-generation fields.

We have calculated these parameters, which are much larger than in the previous paper [1], for  $10^6$  model points for the  $E_6 \times SU(2)_F$  model. The results are shown in Fig. 2, in which the darker region represents a larger density of model points. The model points for the  $E_6$  model without  $SU(2)_F$ , which were calculated in the previous paper [1], are dotted in the figure. The region in which both  $R_1$  and  $R_2$  are small is allowed in the  $E_6 \times SU(2)_F$  model but seems not to be allowed in the  $E_6$  model without  $SU(2)_F$ . Of course, this can happen because the predictions of the two models are different. However, it is also plausible that the allowed region in the  $E_6 \times SU(2)_F$  model is included in the allowed region in the  $E_6$  model without  $SU(2)_F$  if more model points are taken into account. Therefore, we have recalculated the allowed region by using 100 times more model points for the  $E_6$  model without  $SU(2)_F$  (see Fig. 3). The allowed region for the  $E_6 \times SU(2)_F$  model is almost included in the allowed region for the  $E_6$  model without  $SU(2)_F$ , though the small region with small  $R_2$  is still not included. Since it has been found that increasing the model points is important, in the next subsection we re-examine the conditions for identification of the grand unification group, which were discussed in the previous paper, with 100–1000 times more model points.

We make two comments on Figs. 2 and 3. First, in the  $E_6 \times SU(2)_F$  model, the maximal values of  $R_1$  and  $R_2$  become smaller than those in the  $E_6$  model without  $SU(2)_F$ . This is because the minimum value of  $\Gamma_{p \rightarrow \pi^0 + e^c}$  becomes larger as in Fig. 1 due to the small mixings between the electron and the other charged leptons, as we mentioned in the previous subsection. Second, the minimum value of  $R_2$  becomes smaller than that in the  $E_6$  models, while the minimum value of  $R_1$  does not change so much. This is because the upper bound of the predicted lifetime of the  $p \rightarrow K^0 + \mu^c$  mode in the  $E_6 \times SU(2)_F$  model becomes larger (and therefore the partial decay width becomes smaller), as discussed in the previous subsection, while the upper bound of the predicted lifetime of  $n \rightarrow \pi^0 + \nu^c$  does not change so much.



**Fig. 3.** Contour plot of model point density for the  $E_6$  model without  $SU(2)_F$ . The model point density is defined by the number of model points per unit area  $(\Delta R_1, \Delta R_2) = (0.06, 0.012)$  in the  $(R_1, R_2)$  plain after generating  $10^6$  model points.  $10^4$  model points, which are calculated in Ref. [1], are dotted. VEVs are taken as  $x = 1 \times 10^{16}$  GeV,  $v_c = 5 \times 10^{14}$  GeV, and  $v_\phi = 5 \times 10^{15}$  GeV.

#### 4.3. Identification of GUT models

In this subsection we re-examine the conditions for identification of the grand unification group by using  $10^6$  model points, 100–1000 times more than in the previous paper [1].

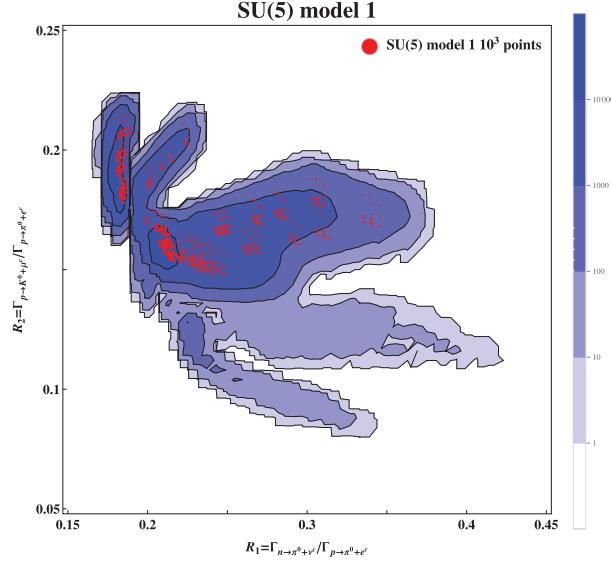
In order to examine the statement that the unification group is not  $SU(5)$  if  $R_1 > 0.4$ , we have calculated  $R_1$  and  $R_2$  in the  $SU(5)$  model with  $10^6$  model points (see Fig. 4). The figure shows that there are very few model points with  $R_1 > 0.4$ . Therefore, the statement is almost satisfied even if  $10^6$  model points are taken into account.

In order to examine the statements that the unification group is  $E_6$  if  $R_1 > 1$  and that the unification group is implied to be  $E_6$  if  $R_2 > 0.3$ , we have calculated  $R_1$  and  $R_2$  in the  $SO(10)$  model with  $10^6$  model points (see Fig. 5). Note that the effect of the  $SO(10)$   $X$ -type gauge boson  $X'$  becomes almost maximal with the VEVs adopted in the calculation. The figure shows that there are very few model points with  $R_1 > 1$  or  $R_2 > 0.3$ . Therefore, these statements are almost satisfied even if  $10^6$  model points are taken into account.

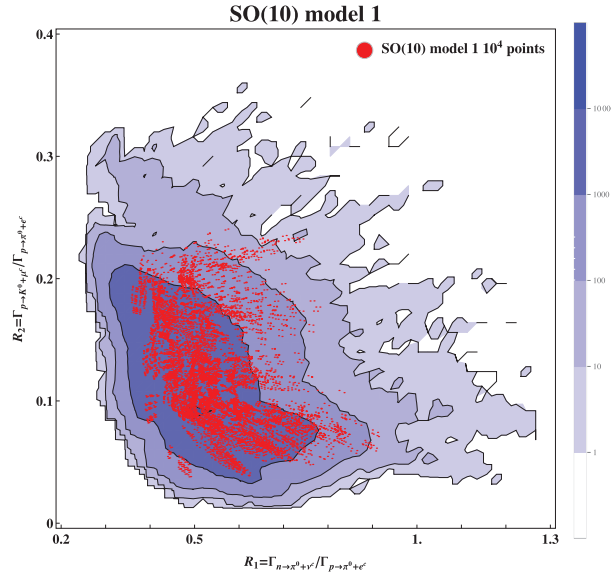
At the end of this subsection, we show the result in the  $E_6 \times SU(2)_F$  model with  $x = \frac{2}{3} \times 10^{16}$  GeV. This is nothing but Choice B in Appendix B to obtain our Yukawa matrices. The difference is only the VEV of the adjoint Higgs. Roughly, the lifetime becomes  $(2/3)^4$  times shorter. As seen in Fig. 6,  $E_6 \times SU(2)_F$  with smaller  $x$  predicts smaller  $R_1$  and  $R_2$  than the original  $E_6 \times SU(2)_F$  model, which has  $x = 1 \times 10^{16}$  GeV. This is because the nucleon decay via dimension-6 operators that is induced by  $X''$  exchange is less significant in the  $E_6 \times SU(2)_F$  model with smaller  $x$  than the nucleon decay induced by  $X$  exchange.

## 5. Discussion and summary

In this paper we have calculated the partial lifetime for various decay modes of nucleons via dimension-6 operators in the anomalous  $U(1)_A$   $E_6 \times SU(2)_F$  SUSY GUT model with spontaneously broken CP symmetry. Once we fix the VEVs of the GUT Higgs, the main ambiguities come

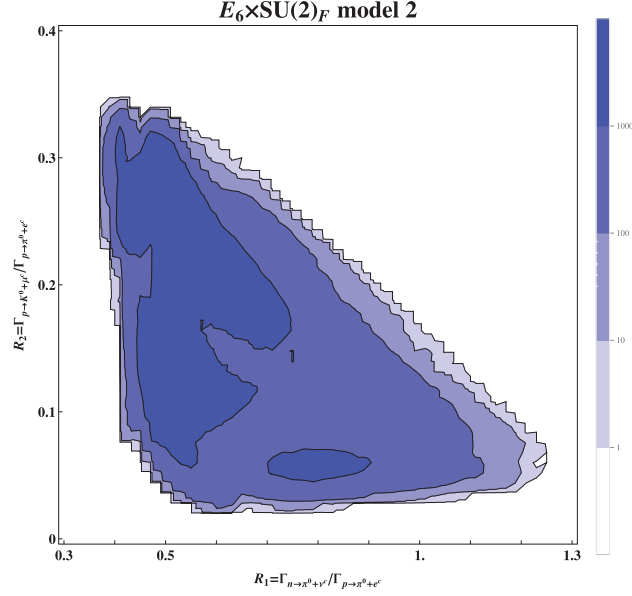


**Fig. 4.** Contour plot of  $SU(5)$  model point density. The model point density is defined by the number of model points per unit area  $(\Delta R_1, \Delta R_2) = (0.006, 0.004)$  in the  $(R_1, R_2)$  plain after generating  $10^6$  model points.  $10^3$  model points, which are calculated in Ref. [1], are dotted. VEVs are taken as  $x = 1 \times 10^{16}$  GeV.



**Fig. 5.** Contour plot of  $SO(10)$  model point density. The model point density is defined by the number of model points per unit area  $(\Delta R_1, \Delta R_2) = (0.022, 0.008)$  in the  $(R_1, R_2)$  plain after generating  $10^6$  model points.  $10^4$  model points, which are calculated in Ref. [1], are dotted. VEVs are taken as  $x = 1 \times 10^{16}$  GeV,  $v_c = 5 \times 10^{14}$  GeV.

from the diagonalizing matrices of quark and lepton mass matrices. Since the  $SU(2)_F$  symmetry can reduce the ambiguities, the predictions have become more restricted than the  $E_6$  model without  $SU(2)_F$  symmetry. We have derived the various relations between the components of the diagonalizing matrices from the constraints on the Yukawa couplings that are realized in the  $E_6 \times SU(2)_F$  model. Among these relations, we have used 9 relations that are not dependent on the renormalization scale. We have shown that only 6 parameters are sufficient to fix the 7 diagonalizing  $3 \times 3$  matrices.



**Fig. 6.** Contour plot of the model point density of  $E_6 \times SU(2)_F$  model 2. The model point density is defined by the number of model points per unit area  $(\Delta R_1, \Delta R_2) = (0.020, 0.008)$  in the  $(R_1, R_2)$  plain after generating  $10^6$  model points. VEVs are taken as  $x = \frac{2}{3} \times 10^{16}$  GeV,  $v_c = 5 \times 10^{14}$  GeV, and  $v_\phi = 5 \times 10^{15}$  GeV.

In this calculation, we have increased the model points up to  $10^6$  from  $10^3$ – $10^4$  in the previous paper. Even with so many model points, the previous conclusion is still valid, that  $R_1 = \frac{\Gamma_{n \rightarrow \pi^0 + \nu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}}$  and  $R_2 = \frac{\Gamma_{p \rightarrow K^0 + \mu^c}}{\Gamma_{p \rightarrow \pi^0 + e^c}}$  are useful to identify the grand unification groups,  $SU(5)$ ,  $SO(10)$ , or  $E_6$ , in the anomalous  $U(1)_A$  GUTs.

It is important to consider how to test the GUT models. The most important prediction of the GUT is nucleon decay and, therefore, the calculations for the partial decay widths for various GUT models are important. One more interesting piece of evidence of the GUT models may appear in the SUSY breaking parameters, especially in scalar fermion masses through the  $D$ -term contribution, which are generated if the rank of the unification group is larger than 4 or an additional gauge symmetry like  $SU(2)_F$  is introduced [47–55]. We will study this possibility in the  $E_6 \times SU(2)_F$  models in future. The estimation of the diagonalizing matrices in this paper must be important in predicting the FCNC processes induced by the non-vanishing  $D$ -term.

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### Appendix A. The diagonalization of Yukawa matrices (in leading order)

Hereafter, we summarize how to diagonalize the  $3 \times 3$  matrix  $Y_{ij}$ . In this calculation we suppose that the Yukawa matrix has hierarchies,  $Y_{ij} \ll Y_{kj}$  and  $Y_{ij} \ll Y_{il}$  when  $i < k$  and  $j < l$ . For this calculation see Refs. [30,56].



Diagonalizing the Yukawa matrix, we translate the flavor eigenstate  $\psi$  into the mass eigenstate  $\psi'$ . We make the Yukawa matrix  $Y$  diagonal, as

$$\begin{aligned}\psi_{Li} Y_{ij} \psi_{Rj}^c &= (L_\psi^\dagger \psi_L)_i (L_\psi^T Y R_\psi)_{ij} (R_\psi^\dagger \psi_R^c)_j \\ &= \psi'_{Li} Y_{\text{diag } ij} \psi'^c_{Rj},\end{aligned}\quad (\text{A1})$$

where unitary matrices  $L_\psi$  and  $R_\psi$  are the diagonalizing matrices, and  $i, j$  ( $i, j = 1, 2, 3$ ) are the indices of generation.

We express the parameters for the diagonalizing matrices  $L$  and  $R$  as

$$L^T \equiv \begin{pmatrix} c_{12}^L & -s_{12}^L & 0 \\ s_{12}^{L*} & c_{12}^L & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13}^L & 0 & -s_{13}^L \\ 0 & 1 & 0 \\ s_{13}^{L*} & 0 & c_{13}^L \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23}^L & -s_{23}^L \\ 0 & s_{23}^{L*} & c_{23}^L \end{pmatrix} \equiv P_{12}^L P_{13}^L P_{23}^L, \quad (\text{A2})$$

$$R \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23}^R & s_{23}^R \\ 0 & -s_{23}^{R*} & c_{23}^R \end{pmatrix} \begin{pmatrix} c_{13}^R & 0 & s_{13}^R \\ 0 & 1 & 0 \\ -s_{13}^{R*} & 0 & c_{13}^R \end{pmatrix} \begin{pmatrix} c_{12}^R & s_{12}^R & 0 \\ -s_{12}^{R*} & c_{12}^R & 0 \\ 0 & 0 & 1 \end{pmatrix} \equiv P_{23}^{R\dagger} P_{13}^{R\dagger} P_{12}^{R\dagger}, \quad (\text{A3})$$

where  $s_{ij}^{L/R} \equiv \sin \theta_{ij}^{L/R} e^{i\chi_{ij}^{L/R}}$  and  $c_{ij}^{L/R} \equiv \cos \theta_{ij}^{L/R}$ . We define the Yukawa matrix  $Y$  as

$$Y \equiv \begin{pmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{pmatrix}. \quad (\text{A4})$$

The Yukawa matrix is diagonalized as

$$\begin{aligned}L^T Y R &= P_{12}^L P_{13}^L P_{23}^L \begin{pmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \\ y_{31} & y_{32} & y_{33} \end{pmatrix} P_{23}^{R\dagger} P_{13}^{R\dagger} P_{12}^{R\dagger} \\ &\simeq P_{12}^L P_{13}^L \begin{pmatrix} y_{11} & y'_{12} & y_{13} \\ y'_{21} & y'_{22} & 0 \\ y_{31} & 0 & y_{33} \end{pmatrix} P_{13}^{R\dagger} P_{12}^{R\dagger} \\ &\simeq P_{12}^L \begin{pmatrix} y'_{11} & y'_{12} & 0 \\ y'_{21} & y'_{22} & 0 \\ 0 & 0 & y_{33} \end{pmatrix} P_{12}^{R\dagger} = \begin{pmatrix} y''_{11} & 0 & 0 \\ 0 & y'_{22} & 0 \\ 0 & 0 & y_{33} \end{pmatrix}. \quad (\text{A5})\end{aligned}$$

In the calculation, we use the approximation that the mixing angles are small, i.e.,  $|s_{ij}^{L/R}| \sim |\theta_{ij}| \ll 1$  ( $s_{ij}^{L/R} \sim \theta_{ij} e^{i\chi_{ij}^{L/R}}$ ) and  $c_{ij}^{L/R} \simeq 1$ . The mixing angles of the diagonalizing matrix and eigenvalues are estimated in this assumption as

$$y'_{22} \simeq y_{22} - \frac{y_{23}y_{32}}{y_{33}}, \quad y'_{12} \simeq y_{12} - \frac{y_{13}y_{32}}{y_{33}}, \quad y'_{21} \simeq y_{21} - \frac{y_{23}y_{31}}{y_{33}}, \quad (\text{A6})$$

$$y'_{11} \simeq y_{11} - \frac{y_{13}y_{31}}{y_{33}}, \quad y''_{11} \simeq y'_{11} - \frac{y'_{12}y'_{21}}{y'_{22}}. \quad (\text{A7})$$

$$s_{23}^L \simeq \frac{y_{23}}{y_{33}}, \quad s_{13}^L \simeq \frac{y_{13}}{y_{33}}, \quad s_{12}^L \simeq \frac{y_{12}y_{33} - y_{13}y_{32}}{y_{22}y_{33} - y_{23}y_{32}}. \quad (\text{A8})$$

$$s_{23}^{R*} \simeq \frac{y_{32}}{y_{33}}, \quad s_{13}^{R*} \simeq \frac{y_{31}}{y_{33}}, \quad s_{12}^{R*} \simeq \frac{y_{21}y_{33} - y_{31}y_{23}}{y_{22}y_{33} - y_{23}y_{32}}. \quad (\text{A9})$$

For reference, we explain the diagonalization for the  $2 \times 2$  matrix without approximation:

$$\begin{pmatrix} c_L & -s_L \\ s_L^* & c_L \end{pmatrix} \begin{pmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} c_R & s_R \\ -s_R^* & c_R \end{pmatrix} = \begin{pmatrix} y'_{11} & 0 \\ 0 & y'_{22} \end{pmatrix}. \quad (\text{A10})$$

As above, we define that  $s_{L/R} \equiv \sin \theta_{L/R} e^{i\chi_{L/R}}$  and  $c_{L/R} \equiv \cos \theta_{L/R}$ .  $\tan 2\theta_{L/R}$  are taken as

$$\tan 2\theta_L = \frac{2(y_{12}y_{22} + y_{11}y_{21}e^{2i\chi_R})}{y_{22}^2 e^{i\chi_L} - y_{11}^2 e^{-i(\chi_L - 2\chi_R)} + y_{21}^2 e^{i(\chi_L + 2\chi_R)} - y_{12}^2 e^{-i\chi_L}}, \quad (\text{A11})$$

$$\tan 2\theta_R = \frac{2(y_{11}y_{12} + y_{21}y_{22}e^{2i\chi_L})}{y_{22}^2 e^{-i(\chi_R - 2\chi_L)} - y_{11}^2 e^{i\chi_R} - y_{21}^2 e^{i(\chi_R + 2\chi_L)} + y_{12}^2 e^{-i\chi_R}}, \quad (\text{A12})$$

and the eigenvalues become

$$y'_{11} = y_{12}c_L \left( \frac{y_{11}}{y_{12}}c_R - s_R^* \right) - y_{22}s_L \left( \frac{y_{21}}{y_{22}}c_R - s_R^* \right), \quad (\text{A13})$$

$$y'_{22} = y_{12}s_L^* \left( \frac{y_{11}}{y_{12}}s_R + c_R \right) + y_{22}c_L \left( \frac{y_{21}}{y_{22}}s_R + c_R \right). \quad (\text{A14})$$

When the  $2 \times 2$  matrix has a hierarchy,  $y_{11} \ll y_{12} \sim y_{21} \ll y_{22}$ , the angles and eigenvalues are approximately obtained as

$$s_L^* \sim \frac{y_{12}}{y_{22}} e^{-i\chi_L}, \quad s_R \sim \frac{y_{21}}{y_{22}} e^{i\chi_R}, \quad (\text{A15})$$

$$y'_{11} \sim y_{11} + \frac{y_{12}y_{21}}{y_{22}}, \quad y'_{22} \sim y_{22}. \quad (\text{A16})$$

The diagonalizing matrices of the left-handed up-type quark and down-type quark,  $L_u$  and  $L_d$ , are given by

$$L_{u/d}^T = P_{12}^{u/dL} P_{13}^{u/dL} P_{23}^{u/dL} \simeq \begin{pmatrix} 1 & -s_{12}^{u/dL} & -s_{13}^{u/dL} + s_{23}^{u/dL} s_{12}^{u/dL} \\ s_{12}^{u/dL*} & 1 & -s_{23}^{u/dL} \\ s_{13}^{u/dL*} & s_{23}^{u/dL*} & 1 \end{pmatrix}. \quad (\text{A17})$$

Therefore, the CKM matrix  $U_{\text{CKM}}$  is calculated as

$$\begin{aligned} U_{\text{CKM}} &\equiv L_u^\dagger L_d \\ &\simeq \begin{pmatrix} 1 & s_{12}^{dL} - s_{12}^{uL} & s_{13}^{dL} - s_{13}^{uL} - s_{12}^{uL}(s_{23}^{dL} - s_{23}^{uL}) \\ s_{12}^{uL*} - s_{12}^{dL*} & 1 & s_{23}^{dL} - s_{23}^{uL} \\ s_{13}^{uL*} - s_{13}^{dL*} - s_{12}^{dL*}(s_{23}^{uL*} - s_{23}^{dL*}) & s_{12}^{uL*} - s_{12}^{dL*} & 1 \end{pmatrix} \\ &\equiv \begin{pmatrix} 1 & U_{us} & U_{ub} \\ -U_{us}^* & 1 & U_{cb} \\ U_{us}^* U_{cb}^* - U_{ub}^* & -U_{cb}^* & 1 \end{pmatrix}. \end{aligned} \quad (\text{A18})$$

We can also calculate the MNS matrix by replacement,  $u \leftrightarrow \nu$  and  $d \leftrightarrow e$ .

## Appendix B. Detailed explanation of the coefficients for 1–2 components of Yukawa matrices

In this appendix, we explain why we take the coefficients of the terms with the adjoint Higgs fields  $A$ , whose VEV is given by

$$\langle A(\mathbf{45}) \rangle = i\sigma_2 \times \begin{pmatrix} x & & & \\ & x & & \\ & & x & \\ & & & 0 \\ & & & & 0 \end{pmatrix}, \quad (\text{B1})$$

as the  $O(1)$  coefficient times  $Q_{B-L}$ . First of all, we have to mention that no one knows the correct normalization of the operators that have  $O(1)$  coefficients. Therefore, some ambiguities are inevitable in our arguments. In the following discussions, we use  $SO(10)$  language because it is very familiar.

First, let us summarize the situation. The field content that we have to consider here is matter fields  $\Psi_a(\mathbf{16})$ , an adjoint field  $A(\mathbf{45})$ , and Higgs fields  $\Phi(\mathbf{10})$ . Here  $a = 1, 2$  is the index of  $SU(2)_F$ . Note that  $\epsilon_{ab}\Psi_a\Psi_b\Phi$  is forbidden because of the antisymmetric feature of  $\epsilon_{ab} = -\epsilon_{ba}$ . Since  $\mathbf{16} \times \mathbf{16} = \mathbf{10}_S + \mathbf{120}_A + \mathbf{126}_S$ , only  $\mathbf{120}_A$  is allowed. As a result, all operators  $\epsilon_{ab}\Psi_a\Gamma_{[ABC]}\Psi_b$ ,  $A_{DE}$ ,  $\Phi_F$ , and  $\epsilon_{ABCDEFGH IJ}$  have vector indices of  $SO(10)$  ( $A, B, C, D, E, F, G, H, I, J = 1, 2, \dots, 10$ ). We would like to classify the  $SO(10)$ -invariant operators that contribute to Yukawa couplings. For this purpose, the following observations are important.

1. The indices of  $A_{DE}$  must be colored (i.e.,  $E, D = 1, 2, 3, 6, 7, 8$ ), because the VEV of  $A$  in Eq. (B1) only has support for colored indices.
2. The index of  $\Phi_F$  must be non-colored (i.e.,  $F = 4, 5, 9, 10$ ), because the field must become the MSSM Higgs.
3. The indices of  $\epsilon_{ab}\Psi_a\Gamma_{[ABC]}\Psi_b$ ,  $A_{DE}$ , and  $\epsilon_{ABCDEFGH IJ}$  must be totally antisymmetric.

The first two observations lead to a rule that the indices of  $A$  and  $\Phi$  must not be contracted. Then only two kinds of operators invariant under  $SO(10)$  are allowed as

$$\begin{aligned} & \epsilon_{ab}\Psi_a\Gamma_{[ABC]}\Psi_b\Phi_A(A^n)_{BC}, \\ & \epsilon_{ABCDEFGH IJ}\Phi_A\epsilon_{ab}\Psi_a\Gamma_{[BCD]}\Psi_b(A^n)_{EF}(A^m)_{GH}(A^p)_{IJ}, \end{aligned} \quad (\text{B2})$$

where  $n, m, p$  are odd integers and  $\Gamma_{[ABC]}$  is the multiplicity of three gamma matrices of  $SO(10)$  with total antisymmetry on the indices  $A, B, C$ . Strictly speaking, these operators can be multiplied by any polynomial function of  $\text{tr } A^n$  ( $n = \text{even integer}$ ). The first operators induce the Yukawa couplings proportional to  $Q_{B-L}$ , while the second operators contribute to universal Yukawa coupling. Therefore, if:

Choice A The contribution of the second operators that contain a totally antisymmetric tensor  $\epsilon$  is suppressed (or dominated by the contribution of the first operators),

the contribution to the Yukawa couplings becomes roughly proportional to the  $B - L$  charge.

Let us try another explanation. The operator  $A_{AB}\Gamma_{AB}\Psi_a$  gives  $\frac{3}{2}Q_{B-L}\Psi_a$ , and  $(A_{AB}\Gamma_{AB})^n\Psi_a$  gives  $(\frac{3}{2}Q_{B-L})^n\Psi_a$ , under the VEV in Eq. (B1) with  $x = \lambda^{-a}\Lambda$ , where  $a$  is the anomalous  $U(1)_A$  charge of  $A$ , although  $n$  must be odd because of the antisymmetric feature of  $\Psi_a\Psi_b$ . Here,  $\Gamma_{AB}$  is a multiplicity of two gamma matrices  $\Gamma_A$  of  $SO(10)$  as  $\Gamma_{AB} \equiv \frac{1}{2!}(\Gamma_A\Gamma_B - \Gamma_B\Gamma_A)$ . The coefficient  $(\frac{3}{2}Q_{B-L})^n$  from such terms  $A^n\Psi$  can be much larger than 1 when  $n$  is larger. This means that the predictions become strongly dependent on the details of the model (explicitly, on the maximum of  $n$

allowed in the model). To avoid this situation, we would like to require that the maximal coefficient from  $A$ , namely the coefficient for the leptons, becomes one. Then the coefficient from such terms  $A^n \Psi$  becomes just  $Q_{B-L}^n$ . There are several ways to realize this. For example,

Choice B An additional factor  $2/3$  for each  $A$  appears in every term with  $A$ . It may look to be artificial, but if the kinetic term of  $A$  has an additional factor  $(3/2)^2$ , such a situation can be realized even when all the other terms have  $O(1)$  coefficients.

Choice C  $x = \frac{2}{3}\lambda^{-a}\Lambda$ .

Note that under such requirements, the second operators in Eq. (B2) have a suppression factor  $(2/3)^{n+m+p}$ . Therefore, the expectation that the Yukawa couplings are roughly proportional to  $Q_{B-L}$  becomes reasonable.

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